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If we employ the equations

$$x^2/a^2+y^2/b^2+z^2/c^2=1 \ldots\ldots (4),$$

$$x^2/a^2+y^2/b^2=1 \ldots\ldots (5),$$

$$x^2/a^2+y^2/b^2=z^2/c^2 \ldots\ldots (6),$$

for (1), (2), (3), respectively, the theorem is still true. If for (4) we substitute either of the equations

$$x^2/a^2+y^2/b^2=1+z^2/c^2 \ldots\ldots (7),$$

$$x^2/a^2+y^2/b^2=z^2/c^2-1 \ldots\ldots (8),$$

we get the theorems that the volume of a segment of (7) made by planes parallel to that of x, y is equal to the sum of the volumes of the corresponding segments of (5) and (6); and that of a segment of (7) is the volume of the corresponding segment of (6) diminished by that of the segment of (5).

The volume of a segment of (1) made by planes parallel to that of x, y is equal to the sum of the volumes of two cylinders whose bases are the bases of the segment and altitudes half that of the segment, together with the volume of a sphere to which the bases of the segment are tangent.

The corresponding theorems are as follows: The volume of a segment of (4) made by planes parallel to that of x, y is equal to the sum of the volumes of two cylinders whose bases are the bases of the segment and whose altitudes are half that of the segment, together with the volume of an ellipsoid to which the bases of the segment are tangent, similar to (4) and similarly placed.

Furthermore, the volume of a segment of (7) or (8) made by planes parallel to that of x, y is equal to the volumes of two cylinders whose bases are the bases of the segment, diminished by the volume of an ellipsoid to which the bases are tangent, whose axes are proportional to those of (7) or (8) and respectively parallel to them.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

178. **Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.**

A_n being the arithmetic mean of the nth powers of the numbers less than p and prime to it, find a relation between A_3, A_2 and p.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The number n of positive integers less than p and prime to it is

$$n=p(1-1/a)(1-1/b)(1-1/c)(1-1/d).....$$

The sum of the squares of all such numbers is

$$S_2=\tfrac{1}{3}p^3(1-1/a)(1-1/b)(1-1/c)(1-1/d).....+\tfrac{1}{6}p(1-a)(1-b)(1-c)(1-d).....$$

and the sum of the cubes is

$$S_3=\tfrac{1}{4}p^4(1-1/a)(1-1/b)(1-1/c)(1-1/d).....+\tfrac{1}{4}p^2(1-a)(1-b)(1-c)(1-d).....$$

$$\therefore A_2=S_2/n=\tfrac{1}{3}p^2+\tfrac{1}{6}\cdot\frac{(1-a)(1-b)(1-c)(1-d).....}{(1-1/a)(1-1/b)(1-1/c)(1-1/d).....}$$

$$=\tfrac{1}{3}p^2+\tfrac{1}{6}B, \text{ suppose.}$$

$$A_3=S_3/n=\tfrac{1}{4}p^3+\tfrac{1}{4}pB.$$

$$\therefore B=6A_2-2p^2=\frac{4A_3-p^3}{p}.$$

$$\therefore 6A_2p-2p^3=4A_3-p^3, \text{ or } 6A_2p-4A_3=p^3, \text{ or } p^3-6A_2p+4A_3=0.$$

179. Proposed by DR. L. E. DICKSON, The University of Chicago.

Find the roots of the algebraically solvable quintic equation

$$x^5+qx^2+px+\tfrac{1}{5}\left(\frac{q^2}{p}-\frac{p^3}{5q}\right)=0.$$

No solution of this problem has been received.

180. Proposed by the late JOSIAH H. DRUMMOND.

If r/s is such a value of p as makes $m/(p^2-2)$ integral, prove that $(3r+4s)/(2r+3s)$ is another such value, so that an indefinite number of integral values may be obtained.

Also, if r/s is such a value of p as makes $2m/(p^2-2)$ integral, prove that $2(r+s)/(r+2s)$ is also such a value.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

When $p=r/s$, $m/(p^2-2)=ms^2/(r^2-2s^2).....(1)$.

When $p=(3r+4s)/(2r+3s)$, $m/(p^2-2)=m(2r+3s)^2/(r^2-2s^2).....(2)$.

Since (1) is integral, (2) is also, for we can take $m=n(r^2+2s^2)$.